HelmholtzZentrum münchen
Deutsches Forschungszentrum für Gesundheit und Umwelt

Environmental Modelling for RAdiation Safety II – Working group 9

Comparison between test field data and Gaussian plume model

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HELMHOLTZ
GEMEINSCHAFT

Gaussian dispersion model

Gaussian plume model for concentration in air
[Bq m³]

$$C_r(x,y,z) = \frac{Q_r}{2\pi\sigma_y(x)\sigma_z(x)v(x)} e^{-y^2/2\sigma_y^2(x)} \left(e^{-(z+z_o)^2/2\sigma_z^2(x)} + e^{-(z-z_o)^2/2\sigma_z^2(x)} \right)$$

Pasquill-Gillford coefficients to determine σ_y and σ_z

$$\begin{cases} \sigma_y(x) = ax^{0.894} \\ \sigma_z(x) = cx^d + f \end{cases}$$

z_0 = h + plume rise

$$C_r(x,y,z=0) = \frac{Q_r}{\pi\sigma_y(x)\sigma_z(x)v(x)} e^{-\frac{y^2}{2\sigma_y^2(x)} - \frac{z^2{}_o}{2\sigma_z{}^2(x)}}$$

Deposition velocity v_d

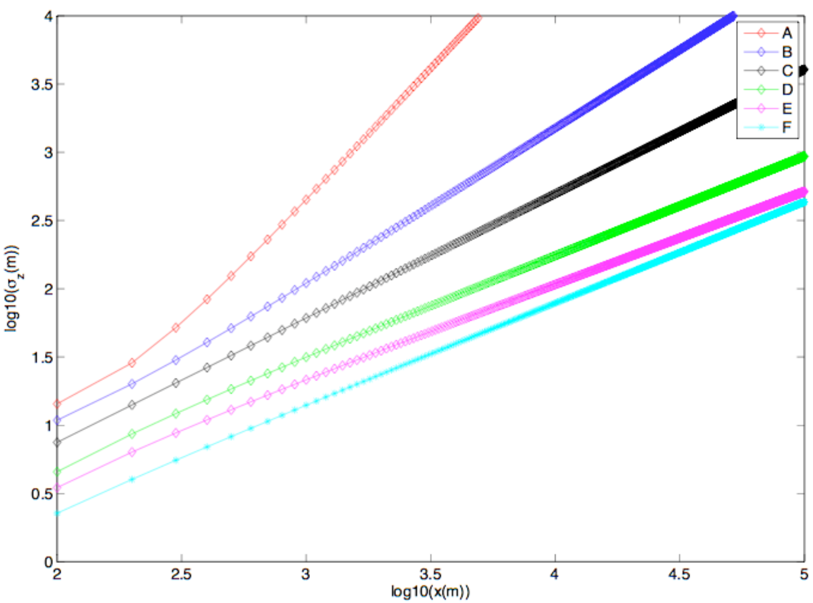
Surface Activity [Bq/m²]

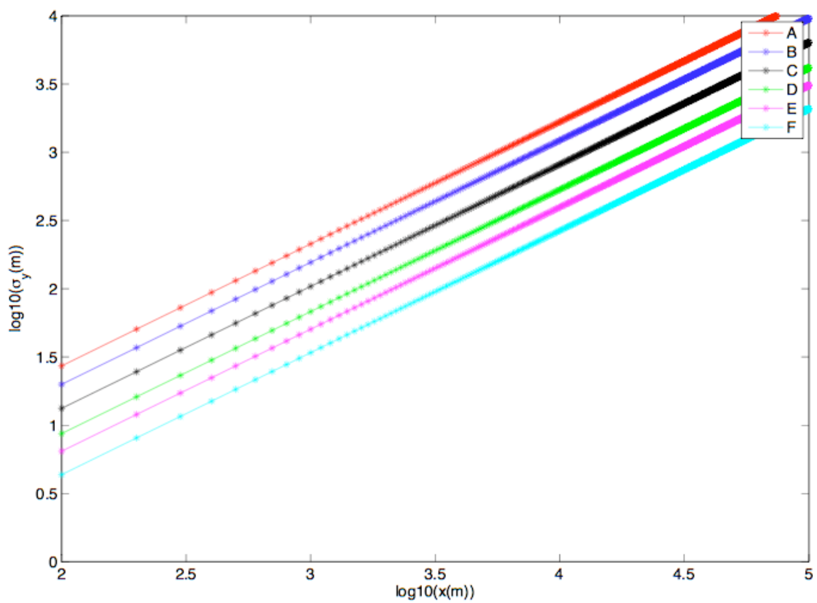
For most materials, a dry deposition velocity of about $1.5*10^{-3}$ m/s can be assumed and the dry deposition flux to the surface (e.g.,B /m2), can be assumed to equal the dry deposition velocity times the concentration. Depletion factor DF neglected (~0.99).

$$B_r(x,y) = v_d \frac{Q_r}{\pi\sigma_y(x)\sigma_z(x)} e^{-\frac{y^2}{2\sigma_y(x)^2}} e^{-\frac{z_0^2}{2\sigma_z(x)^2}} DF(x)$$

STABILITY CLASSES (Martin,1976)

stability	a	c (< 1 km)	d (< 1 km)	f (< 1 km)	c (> 1 km)	d (> 1 km)	f (> 1 km)
A	213	440.8	1.941	927	459.7	2.094	-9.6
B	156	106.6	1.149	3.3	108.2	1.098	2.0
C	104	61	0.911	0	61.0	0.911	0
D	68	33.2	0.725	-1.7	44.5	0.516	-13.0
E	50.5	22.8	0.678	-1.3	55.4	0.305	-34.0
F	34	14.35	0.740	-0.35	62.6	0.180	-48.6





Levenberg-Marquardt algorithm

Non-linear least square fit: minimise the sum of the weighted residuals between measured data $y(t_i)$ and the curve-fitting function $Y(t_i,n)$ where *n* is the number of parameters to be fitted (combination of Gauss-Newton and steepest gradient method)

$$\chi^2(n) = \frac{1}{2}\sum_{i=1}^{m}[y(t_i)^2 - Y(t_i, n)]^2$$

- Iterative improvement to parameter values
- Initial guess for *n* parameters
- MINPACK (fortran90)

Application to experimental data

Fit the experimental data obtained from test1 and test2 to Gaussian plume model and determine unknown parameters such as Q_r, a, c, d, f, z_0, ...

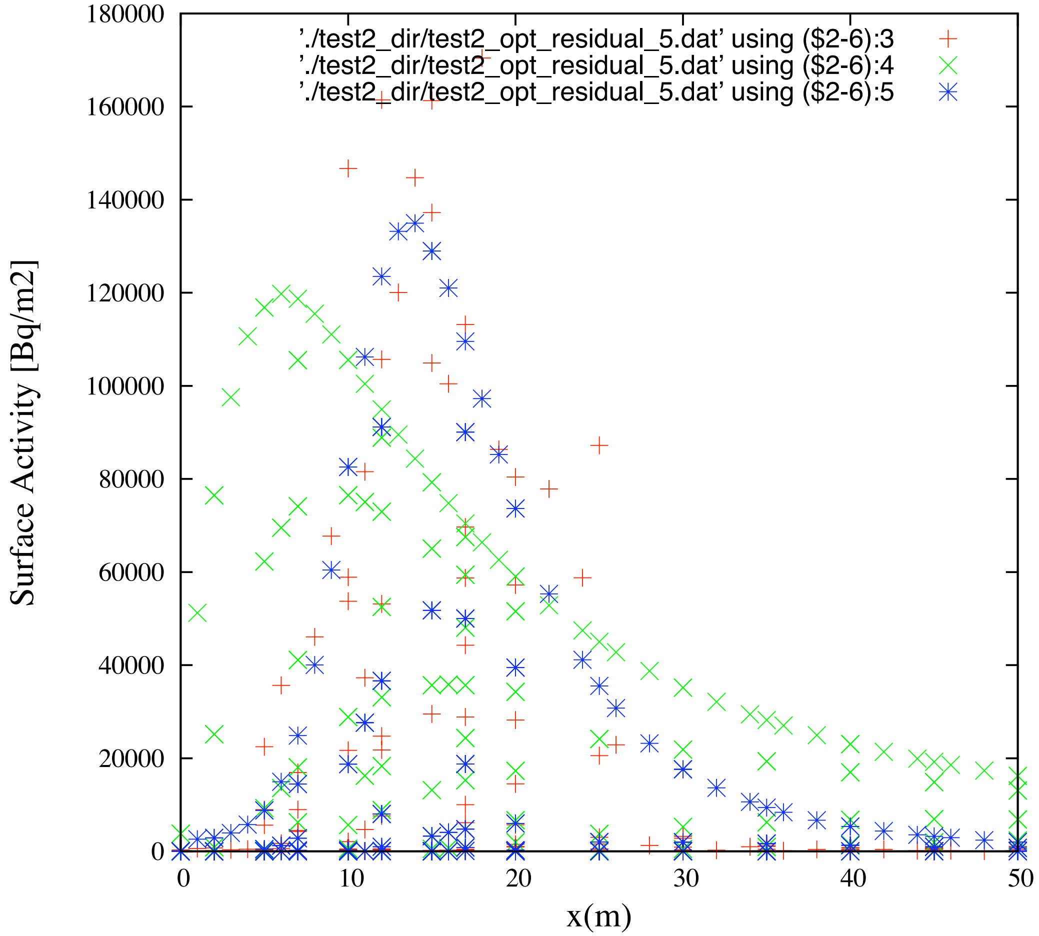
Test 2

Stability class “C”
wind =1.1 m/s (direction parallel to x-axis)

X_0 [a=100 c=60 d=1 f=0 Q_r=1D9]

X_{opt} [a=46 c=6D6 d=4 f=0.3 Q_r=7.3D8]

c is unreasonably high!



Test 2

However, from measurements one can see that in x-direction the plume has a Gaussian shape as well e.g *Diffusion plays a role also in this direction*!

This can be accounted by adding the term $e^{-\frac{(x-vt)^2}{2\sigma_x^2}}$ in the model..
e.g. Gaussian bell with σ_x ~ 6.5 m (from measured data) centred at x_0 = 15 m

RESULTS: (initial guess a=100 c=60 d=1 f=0 Qr=1D9 h=1)

N = 6 a, c, d, f, Q_r, h
N = 5 a, c, d, f, Q_r (h set to 1 m) --> much better than without σ_x
N = 4 a, c, d, f (Q_r set to 1D9 Bq/m^2 and h=1 m) *DIFFICULT TO FIND global MINIMUM, changes immediately by changing initial guess by a ε-difference!*
N = 4 a, c, d (Q_r set to 1D9 Bq/m^2 , h=1 m, f=0)
N = 2 Q_r, h (Stability class C)
N = 1 Q_r convergence not achieved (Stability class C, h=1 m)

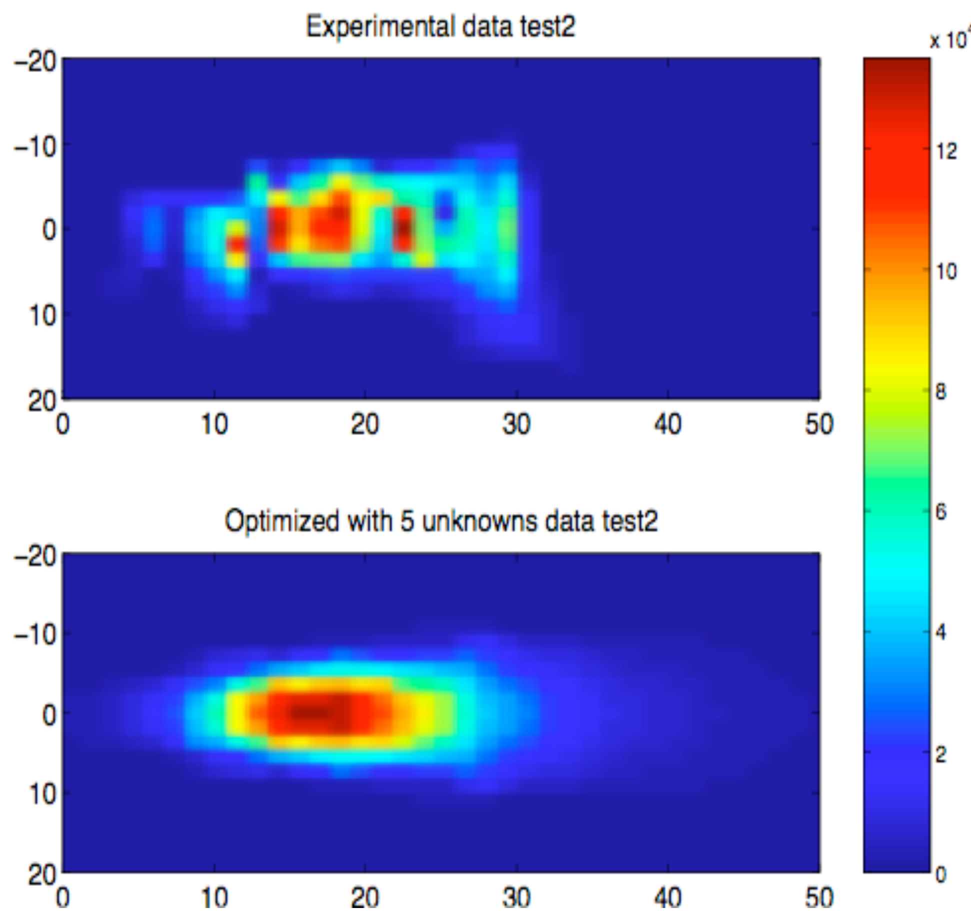
Test 2

Covariance matrix results in strong correlation among some quantities e.g power law dependencies!

Bias in the estimate of uncertainties (standard deviation underestimated)

z_0 = 0 m, 0.5 m, 1 m, 1.5 m makes a big difference in the result of the fit!

Difficult to fit all parameters all together!



Test 2

N = 6 : a, c, d, f, $Q_{r,}$ h
6.40 138.49 0.81 -1.75 3.74D9 5.08
RESNORM = 217370.50 corr= 0.8964595719

N = 5 : a, c, d, f, Q_r
46.40 27.10 0.809 -0.34 7.36D8
RESNORM = 217370.41 corr=0.8964597297

This is the best result!

N = 4 : a, c, d, f
47.16 10.46 3.77E-002 -8.43
RESNORM = *219854* *corr=0.8938*

N = 3: a, c, d
47.39 6.72 0.62
RESNORM = 221427 corr=0.8922

N = 2: Q_r, h
1.88D9 1.62
RESNORM =303304 corr=0.79

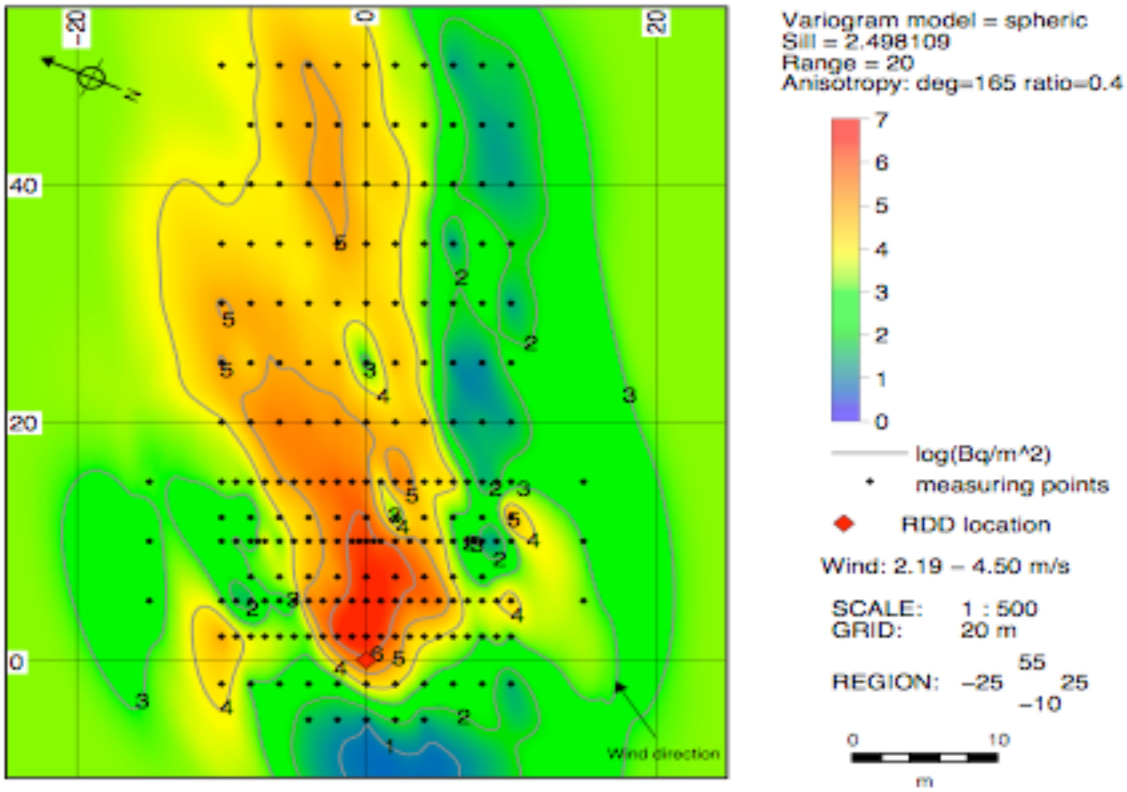
N = 1 : Q_r convergence NOT achieved! (number of maximum iterations reached)
9.5D8
RESNORM = 325253

Summary

- Gaussian plume model applied to test2 (221 points)
- Levenberg-Marquardt algorithm implemented with MINPACK to fit the data
- stability classes considered based on wind velocity and weather characteristics (but also on the initial guess that fits better)
- correlation study carried out
- fit of test2 gives reasonable results by using stability class C!
  However need to change the model by allowing for diffusion in the x-direction!

- Gaussian plume model: to what extent can we apply it to an explosion?
  Stack height z_0 is too simplistic, release time is δ-function
- Stability classes: how uncertain are they? is there a better way to fit data to stability classes e.g avoid fitting a, c, d, f separately?
- how many parameters all together can we fit? seems 5
- what is the minimum number of points necessary to fit with reasonable accuracy?
  down to ~100 but then slow convergence....
- test1 angle fit is also necessary for wind direction

Test 1



Variogram model = spheric
Sill = 2.498109
Range = 20
Anisotropy: deg=165 ratio=0.4
log(Bq/m^2)
measuring points
RDD location
Wind: 2.19 – 4.50 m/s
SCALE: 1 : 500
GRID: 20 m
REGION: –25 55 25 –10
Wind direction
m